Chapter 1

SOLVING FOR A VARIABLE

When the term equality is used – it means that there an equal sign in the equation. The only important thing to remember is **to keep the equal sign the same math operation must be done to each side.**

Another name for equal is the same. When using the properties think of doing the same to each side of the equation. $2+1=3$

$2+1$ **is the same as** 3

If we add the same number to each side the equation is still true.

the same as

$$2+1+6=3+6$$

Because 6 was added to each side, the equation stayed true. Before it was $3=3$ and now it is $9=9$.

Now look at subtracting from each side.

$$4=3+1$$

Simplified it is:

$$4=4$$

If 2 is subtracted from each side, it stays equal.

$$4-2=3+1-2$$

the same

Simplified it is:

$$2=2$$

Two was subtracted from each side of the equation. The new equation solved is $2=2$. The equation remains true.

These are the steps used to solve equations with a variable. The sole purpose right now is to get the variable alone – called isolating the variable. Think of variables as “not playing nice” with the numbers and they must be separated with the equal sign.

the variable who doesn’t play nice with the numbers:

 $-2 = 17$

the opposite of -2 is $+2$

When 2 is added to each side, the variable can be isolated:

 $-2 = 17$

$+2 \quad +2$

the same

 $-2 = 17$

$+2 \quad +2$

 $= 19$

which simplifies to:

 $= 19$

This is the reason learning the inverse priority is important, because it is now to isolate the variable in equations.

Once the equation is solved for the variable, check the answer. Replace (orsubstitute) the variable with the answer, and check.

Example:

$$\begin{array}{r} x + 7 = 20 \\ -7 \quad -7 \\ \hline x \qquad = 13 \end{array}$$

To check your answer, replace the variable x with 13. **Always put the number you are substituting in parenthesis!**

Original equation: $x + 7 = 20$

$$(13) + 7 = 20$$
$$20 = 20$$

Therefore, your answer is correct.

If the number is substituted into the equation without parenthesis, it becomes confusing if you are adding or multiplying. Look at the following example.

$$\begin{array}{rcr} 2x - 3 &=& -7 \\ +3 && +3 \\ \hline 2x &=& 4 \end{array}$$

$$x = -2$$

Substituting -2 back into the equation with parenthesis: $2(-2) - 3 = 7$

With the -2 in parenthesis, it is obvious it is being multiplied by 2 which is -4.

Without using parenthesis: $2 - 2 - 3 \neq -7$ which would equal -3.

Practice - Solving Linear Algebraic Equations – Intro to Addition & Subtraction

Solve the following equations and check your answers.

1. $\begin{array}{rcr} x - 2 &=& 6 \\ +2 && +2 \\ \hline \end{array}$ Check your answer: $(\_\_\_\_) - 2 = 6$

   $\_\_\_\_ = 6$

2. $\begin{array}{rcr} 2 &=& y + 1 \\ -1 && -1 \\ \hline \end{array}$ Check your answer:

3. $x - 2 = -2$ Check your answer:

4. $y + 2 = 4 - 1$

4a. Combine numbers on each side of the equal sign.

$$y + 2 = \underbrace{4 - 1}_{3}$$

$$y + 2 = 3$$

4b. Solve. Check your answer:

$$y + 2 = 3$$

5. $y + 5 - 2 = -1$

5a. Combine any numbers on each side of the equal sign.

5b. Solve. Check your answer:

6. $2 - 3 = x + 4$

6a. Combine any numbers on each side of the equal sign.

6b. Solve. Check your answer:

7. $6 - 2 = 4 + x - 2$

7a. Combine any numbers on each side of the equal sign.

7b. Solve. Check your answer:

Simplify the following equations and check your answers.
Advanced Problems:

8. $3(x - 2) = 9$ Check your answer:

9. $14 = 2(x + 4)$ Check your answer:

10. $(-2)(x + 7) = 32$ Check your answer:

Another inverse property is for multiplication and division. Use this property like the addition and subtraction in the prior examples. Look at the following equation:

Example:

$$2x = 14$$

2*x* means 2 times *x*

The opposite of multiplication is division.

If both sides are divided by $2,$ the variable will be isolated.

$$\frac{2x}{2} = \frac{14}{2}$$

The 2's cancel each other out to isolate the *x*.

$$\frac{{}^{1}\cancel{2}x}{\cancel{2}} = \frac{\cancel{14}^{7}}{\cancel{2}}$$

$$x = 7$$

The above example was how to use the inverse of multiplication. Now look at the inverse oof division:

$$\frac{x}{3} = 4$$

The fraction $\frac{x}{3}$ is stated as $'x \text{ } over \text{ } 3'$ is the same as saying x divided by $3.$

Solve for x by using the inverse of division – which is multiplication – to isolate the variable.

$$(3)\frac{x}{3} = 4(3)$$

$$(\cancel{3})\frac{x}{\cancel{3}} = 4(3)$$

$$x = 12$$

Simplify (which can also be referred to as “isolating the variable”).

A. $3x = 12$

Step 1. What is the number with the variable? ___________

Step 2. Is the variable being multiplied or divided by that number? _____________

Step 3. What is the inverse operation? _____________

Step 4. Solve.

Original problem: $3x = 12$

Take the number in step 1 and perform the operation from step 3.

____

B. $\frac{x}{7} = 2$

Step 1. What is the number with the variable? ___________

Step 2. Is the variable being multiplied or divided by that number? _____________

Step 3. What is the inverse operation? _____________

Step 4. Solve.

Original problem: $\frac{x}{7} = 2$

Take the number in step 1 and perform the operation from step 3.

____

C. $45 = 5y$ (Do not let the variable being on the other side of the equal sign cause confusion, it is still solved the same way.)

Step 1. What is the number with the variable? ___________

Step 2. Is the variable being multiplied or divided by that number? _____________

Step 3. What is the inverse operation? _____________

Step 4. Solve.

Original problem: $45 = 5y$

Take the number in step 1 and perform the operation from step 3.

____

To solve simple algebraic equations undo addition and subtraction first, then undo multiplication and division.

> Note: sometimes you will come across the terminology "undo addition" which means to subtract. The phrase refers to using the inverse operation.

Example: $2x+5=15$

Look for any addition or subtraction signs.

$$2x+5=15$$

There is an addition sign in this problem. The ***opposite operation of addition is subtraction***. Therefore, subtract 5 from both sides.

$$\begin{aligned} 2x+5&=15 \\ -5\ &\ \ -5 \\ \hline 2x\quad &=10 \end{aligned}$$

There are no more addition or subtraction signs. The $2x$ is the same as saying 2 times x. The 2 and the x are being multiplied together. The ***opposite operation of multiplication is division***.

$$\frac{2x}{2}=\frac{10}{2}$$

$$x=5$$

The final step is to replace 5 for x in the original equation to check your answer.

$$2x+5=15$$

$$\begin{aligned} 2(5)+5&=15 \\ 10+5&=15 \\ 15&=15 \end{aligned}$$

Note: Always put the number you are replacing (or substituting) in parenthesis. If you forget the parenthesis the problem could look like $25+5=15$. This is a common mistake and by simply putting the number you are substituting into the equation in parenthesis that mistake will not be made.

Remember that "replacing" and "substituting" have the same meaning in mathematics.

Simplify:

D. $4x+10=58$

a. Look for any addition or subtraction. Perform the opposite operation.

$$\begin{array}{r} 4x+10= \; 58 \\ -10 \;\; -10 \\ \hline \end{array}$$

Finish the subtraction.

b. Take the new form of the equation from above and look for any multiplication or division needed to be performed to isolate the x.

E. $5y-8=7$

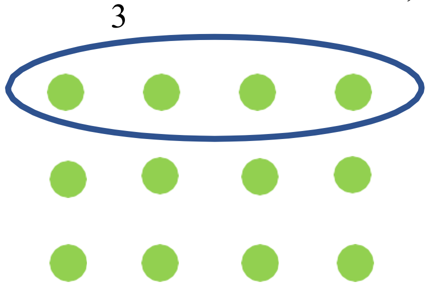
c. Undo any addition or subtraction.

d. Undo any multiplication or division.

Equations with Fractions

The term $\frac{1}{3}x$ is exactly the same as $\frac{x}{3}$

Think about it. If I take $\frac{1}{3}$ of the markers below, I will have 4 markers.



If I divide the 12 markers by 3, I will have 4 markers.

Look at the following examples:

$$\frac{1}{5}y = \frac{y}{5} \qquad \frac{1}{4}k = \frac{k}{4}$$