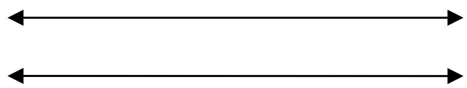
Chapter 3

LINES

Introduction to Parallel Lines

Look at the following lines.

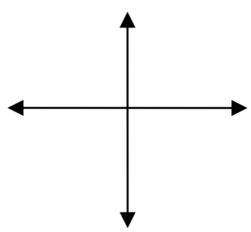


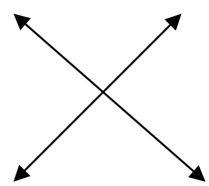
They both have zero slope. They are parallel to each other, they will never ross, and will always stay the same distance from each other.

Parallel Lines must have the same slope!

Introduction to Perpendicular Lines

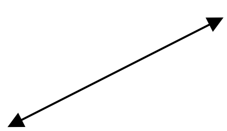
Lines that cross at a 90° angle are perpendicular to each other:



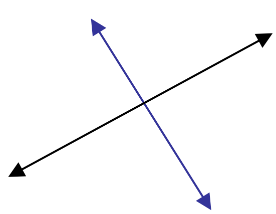


The first set of lines show an angle of zero slope and an undefined slope. They are opposites of each other. One has a slope with a numerator of zero, and the other has a slope with a denominator of zero.

Think about it. What would the slope be of a line that is perpendicular to this line?



This line is a positive slope, so the line perpendicular to it would have to be a negative slope.



And since it is the *exact* *opposite*, **together their slopes would multiply to -1.**

Steps to finding the slope of a line that is perpendicular to it:

1. Find the slope of the line.
2. Determine the multiplicative inverse of the slope.
3. Write the slope of the perpendicular line as the negative inverse of the original slope.

Examples:

Finding the slope of a line perpendicular to a line with a slope of $\frac{2}{3}$.

1. The slope of one of the lines is $\frac{2}{3}$.
2. The opposite (or inverse) of $\frac{2}{3}$ is $\frac{3}{2}$.

   The slope of the line is positive, so a line perpendicular will have a *negative* slope.
3. The slope of a line perpendicular to a line with a slope of $\frac{2}{3}$ is $-\frac{3}{2}$.

Finding the slope of a line perpendicular to a line with a slope of -4.

1. The slope of -4 is negative and is already given.
2. The inverse of 4 is $\frac{1}{4}$.
3. The slope perpendicular to a line with a slope of -4 is $\frac{1}{4}$.

$$-4 \cdot \frac{1}{4} = -1$$

Find the slope of a line that goes through the points (1,2) and (4,11).

1. Determine the slope of the line, using the formula $\frac{y_2 - y_1}{x_2 - x_1} = \frac{11-2}{4-1} = \frac{9}{3} = 3$
2. The inverse of 3 is $\frac{1}{3}$.

The original line has a positive slope, so the perpendicular line will have a negative slope.

3. The slope of the perpendicular line is $-\frac{1}{3}$.

$$3 \cdot \left(-\frac{1}{3}\right) = -1$$

Practice – Lines: Finding the Slope of Parallel And Perpendicular Lines

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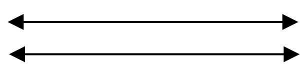
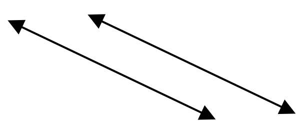
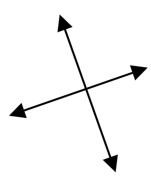
Describe the following lines. Circle the correct answer.

1.  Parallel Perpendicular

2.  Parallel Perpendicular

3.  Parallel Perpendicular

Write the slope of a line.

4. A line parallel to a line with a slope of 2. __________

5. A line perpendicular to a line with a slope of 2. __________

6. A line perpendicular to a line with a slope of -5. __________

7. A line parallel to a line with a slope of -10. __________

8. A line parallel to a line with a slope of $\frac{5}{6}$. __________

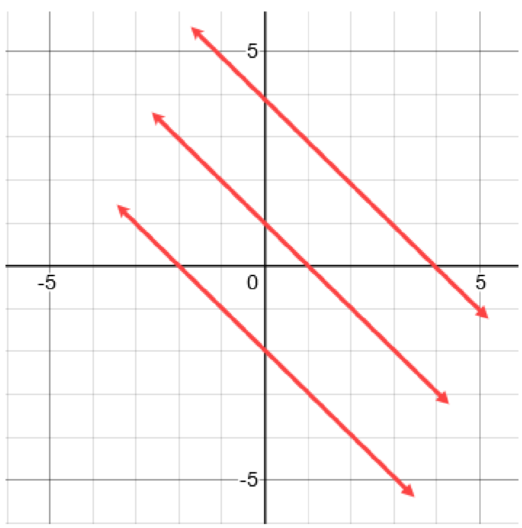
9. A line perpendicular to a line with a slope of $\frac{1}{4}$. ___________

10. A line perpendicular to a line with a slope of $-\frac{3}{4}$.. ___________

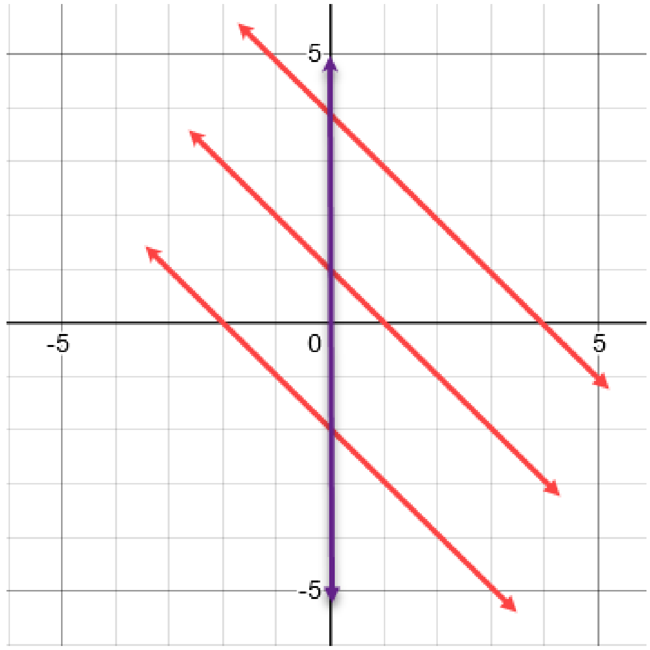
Describing Lines

Lines have slope, and the description the angle of the slope of lines is given by a number. A slope can be a 3 or a -3, and that slope can also be fractions. A slope of 4 will be a steeper slope than a slope of 1. Slopes can also be fractions.

Look at the following lines:



How to tell these lines apart? If the slope was calculated for each of them, all of their slopes would be -1. Notice that they all cross the *y* axis at a different point.



The *y* axis is shown in purple.

The slope for each of these lines is -1. Look at the top line and notice that it crosses the *y* axis at 4.

The format for describing this line is $y = -1x + 4$

slope

Where it crosses the *y*-axis.

Because the slope has a value of 1, it is not necessary to have the 1 written in front of the *x*.

$$y = -1x + 4 \text{ is the same as } y = -x + 4$$

The logic behind this is that it is assumed that we are talking about 1*x* if no number is stated. When you ask for an apple, you would not say, "Can I have *one* apple?" Instead, you might simply say, "Can I have an apple?" It is assumed that you are talking about only one apple. The same concept applies here in math. If there is no number in front of the variable (such as *y* or *x*), it is assumed that the number is 1.

The middle line in the graph above will have the same slope, but a different *y* intercept. It is written as:

$$y = -x + 1$$

slope of 1   *y* intercept of 1.

The bottom line of the graph is:

$$y = -x - 2$$

slope of -1   *y* intercept of -2.

The formula for slope intercept (without numbers) is:

$$y = mx + b$$

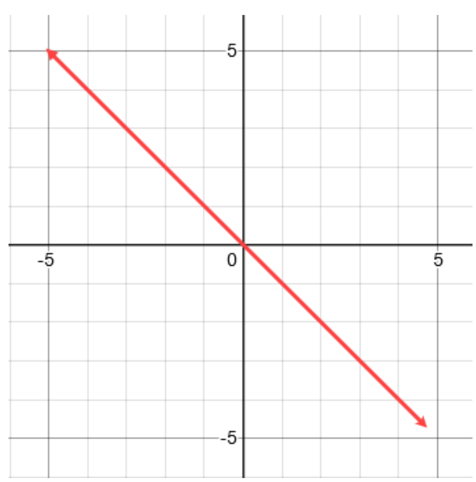
m is the slope

b is the is the *y* intercept

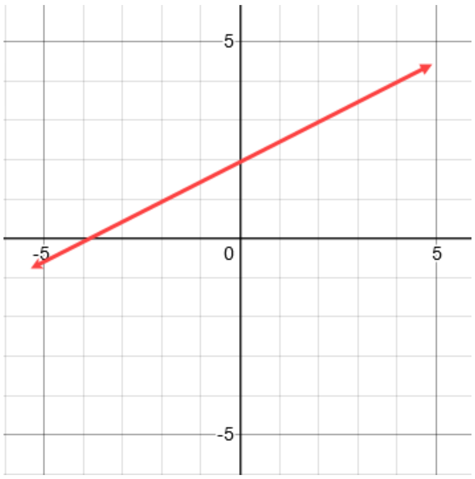
In the above equation: $m = -1 \ \ and \ \ b = -2$

Look back at the graph with the three lines and see that they pass through at 4, 1, and -2.

Look at a line that passes through the middle.
The slope is the same: -1. This is found by picking two points on the line and then using the formula $\dfrac{y_2 - y_1}{x_2 - x_1}$ from chapter 1. The line passes through the *y* axis at zero. It can be written as: $y = -x + 0$, or it is usually simplified as: $y = -x$. There is no need to add the +0



Find the slope of the line and its *y* intercept.



A. The slope is: *m*= ______________ (refer back to chapter 1 if you forgot how to calculate slope)

B. The *y* intercept is: *b*= __________

C. It is written as: *y*= ___*x* + ___

Lines go on forever in each direction (that is why there is an arrow at the end of each line). It is also known how to write a description to tell exactly what that line looks like.

The format is:

$$y = (\text{slope})x + (y - \text{intercept})$$

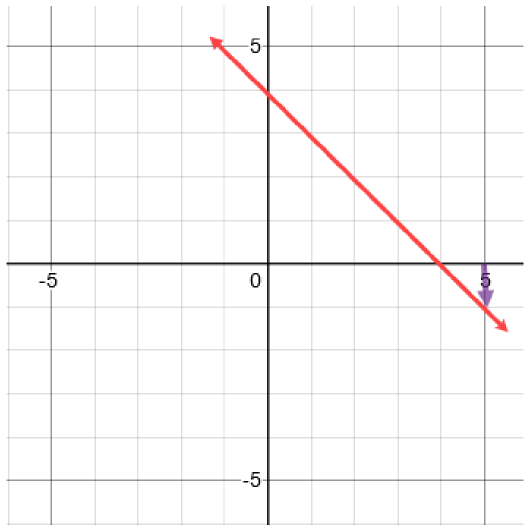
This is simply called slope-intercept form. (Ahaa – it has the *slope* and the *y intercept*,)

Now, look at what can be done with this slope-intercept form. Well, first it is known that lines go on forever in each direction. Therefore, for every *x* value (or number) there is a coordinating *y* value. This means that any value can be put in for *x* to find the corresponding *y* value.

For the first equation $y = -x + 4$ put any value in for *x* and find its corresponding *y* value.

Substitute the number 5 for *x*. The equation would be $y = -5 + 4$.

Solving this: $y = -1$. The line passes through the point $(5, -1)$.



Notice the blue arrow that shows when x is 5 the line's y value is -1. This can be done for any point on any line.

Now try it. If $x = 2$ what is the y value?

Use the equation $y = -x + 4$.

D. The y value is: _______________

Check the answer by looking at the graph on the previous page. Does the line intersect that point? If yes, it is answered correctly.

These problems are easy to plot with the use of an $x \mid y$ chart.

To find points on a line, it is usual to start with the x value. A modified $x \mid y$ chart looks like this.

Place the equation in the middle.

x	$y = -x + 4$	y	(x, y)
5	$y = -5 + 4$	-1	$(5, -1)$

The 5 goes where the x value is

Solving the equation $y = -1$

E. Complete the chart.

Put the equation here ↓

x	$y=-x+4$	y	(x,y)
5	$y=(-5)+4$	-1	$(5,-1)$
1			
-1			
0			

When problems provide the slope-intercept form of an equation, you will be expected to draw the line for that equation. To do this, copy the preceding chart and pick 3 points, then draw the line. The reason why 3 points are chosen is to make sure there was not a mistake on any of the math for the numbers in the equation. If all 3 points line up, it is assumed that the math was performed correctly for those three points.

Pick any three points for *x*. I usually use -2, 0, 2, unless it is a fraction. If the slope is a fraction, it is easiest to use multiples of the denominator. This will give a picture of a positive and negative *x* value.

Note: When x = 0, this gives the y intercept.

F. Complete the chart.

x	$y=2x-3$	y	(x,y)
-2	$y=2(-2)-3$	-7	$(-2,-7)$
0	$y=2(\quad)-3$		(0,__)
2			(2,__)

Draw a line for the equation $y=2x-3$

Look at why this happens. Take the following equations:

$$y = 2x - 12 \qquad y = \frac{1}{4}x + 4 \qquad y = -3x + 1$$

Now substitute 0 for *x* in every equation.

$$y = 2(0) - 12 \qquad y = \frac{1}{4}(0) + 4 \qquad y = -3(0) + 1$$

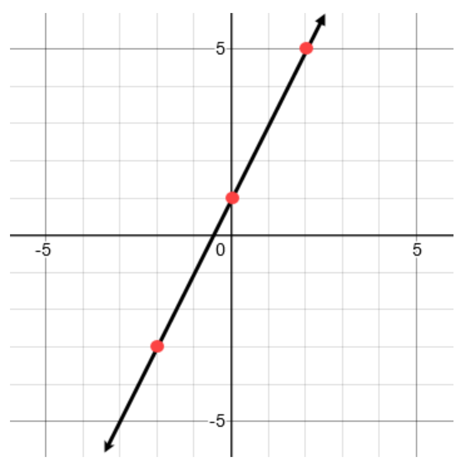
In each equation the slope is being multiplied by 0. Any number multiplied by 0 is 0! The solutions would be:

$$y = -12 \qquad y = 4 \qquad y = 1$$

When you have your *x y* points, you can graph them. (If you need to review, go back to chapter 2.)

For the equation $y = 2x + 1$ the *x y* chart would be:

x	$y = 2x + 1$	y
–2	$y = 2(-2) + 1$	–3
0	$y = 2(0) + 1$	1
2	$y = 2(2) + 1$	5



Practice - Graph a Line from the Linear Equation

Graph the following lines by making an x y chart and plotting the points.

1. $y = 2x + 3$ slope = 2 y intercept = 3

x	$y = 2x + 3$	y
-2	$y = 2(-2) + 3$ $y = -4 + 3$	-1
0	$y = 2(0) + 3$	
2	$y = 2(2) + 3$	

2. $y = -x - 5$ slope = ______ y intercept = ________

x	$y = -x - 5$	y
-2		
0		
2		

Chapter 3 Quiz #1

1. Parallel lines have the same slope.

   True False

2. Perpendicular lines have the same slope.

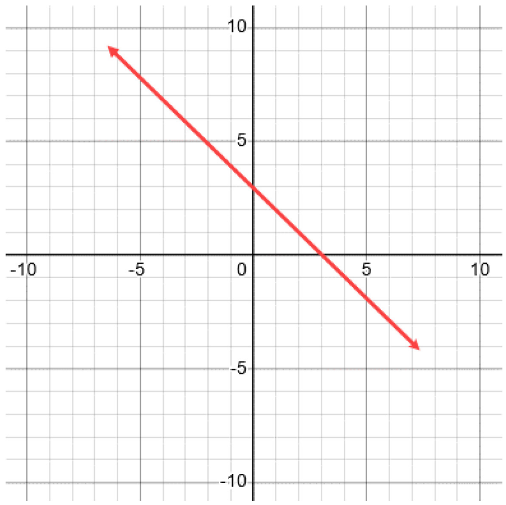
   True False

3. A line with a slope of -5 will have a perpendicular line with a slope of $\frac{1}{5}$.

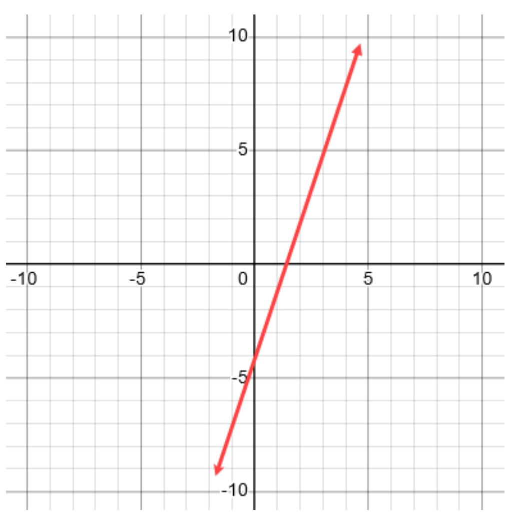
   True False

4. What is the slope of a line perpendicular to $y = 3x - 2$?

5. What is the slope of the line shown?



6. What is the slope of the line shown?



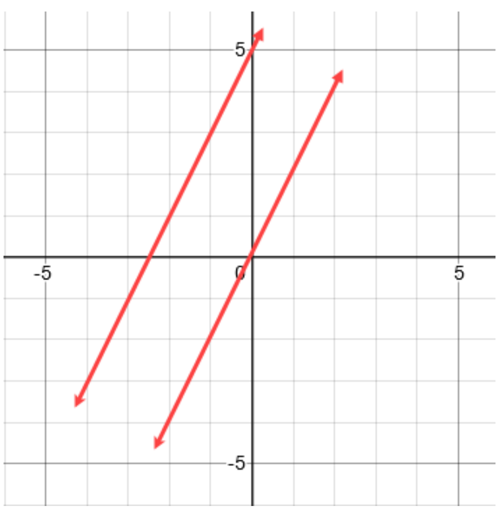
7. Which equation below is a slope-intercept format?
   A. $x = y + 3$
   B. $2x = y - 4$
   C. $y = -3x + 6$
   D. $x + y = 2$
8. For the following equation $y = 5x - 10$
   8a. What is the slope?
   8b. What is the y intercept?

Create and fill in a chart for the following equations. Then graph the equation from the points on the line determined from the graph.

9. $y = 4x - 3$
10. $y = -2x + 3$

Slope of Parallel Lines

How to tell if two lines are parallel? By definition, it is known that parallel lines have the same slope. Look at the two lines below.



The slope for both lines is 2. The difference between these two lines is that they cross at the y axis at different places.

Line 1: $y = 2x + 5$

Line 2: $y = 2x + 0$ or simply $y = 2x$

We do not need to visually see the lines to determine if they are parallel. They will be parallel if their slopes are the same.

Are the following lines parallel?

G. $y = 3x + 2$
$y = 2x + 2$

H. $y = \frac{1}{2}x - 6$
$y = 2x - 4$

I. $y = -2x + 3$
$y = -2x + 4$

Equations may not always come in their simplified form.

Look at the following equations:

$$2y = 2x + 6 \qquad \text{and} \qquad y = x + 4$$

Are they parallel?

To find the answer, each equation must be in the format $\boldsymbol{y =}$.

The first equation is $\boldsymbol{2y =}$ and the second equation is set correctly at $\boldsymbol{y =}$.

Set the first equation to $\boldsymbol{y =}$ by dividing the entire equation by 2. This looks like:

$$\frac{2y}{2} = \frac{6}{2}$$

This will reduce to:

$$y = x + 3$$

The equations in simplified form are:

$$y = x + 3$$
$$and$$
$$y = x + 4$$

Since both equations have just one *x* (remember if there is no number in front of the *x* it means that there is 1) the lines are parallel.

Look at the next set of equations:

$$\frac{1}{2}y = -x + 3 \quad \text{and} \quad 2y = -4x$$

Both equations need to be put into slope-intercept format: $y = mx + b$

(***m*** = slope, ***b*** = *y* intercept)

Take the first equation: $\frac{1}{2}y = -x + 3$

Multiply each of the terms in the equation by 2:

$$(2)\frac{1}{2}y = (-x + 3)(2)$$

The equation in slope-intercept form is: $y = -2x + 6$

The second equation is: $2y = -4x$

Divide both sides by 2: $\frac{2}{2}y = \frac{-4}{2}x$ becomes $y = -2x$

Now, compare the two:

First equation: $y = -2x + 6$

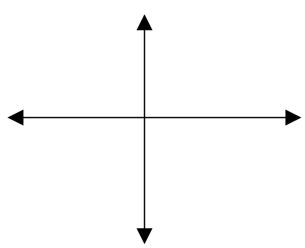
Second equation: $y = -2x$

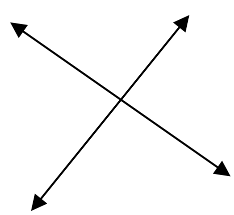
Since the slope on both lines are the same (-2) the lines are parallel. Where does each line cross the y intercept? The first line crosses at $+6$ and the second line crosses at 0.

Note: If they have the same slope and the same y intercept, they are the same line.

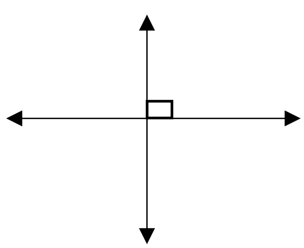
Slope of Perpendicular Lines

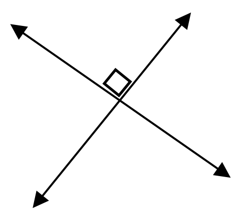
What about perpendicular lines? By definition, perpendicular lines meet at a 90º angle. Look at the examples below:





To show that they are perpendicular, the symbol of a box is shown in one of the corners:

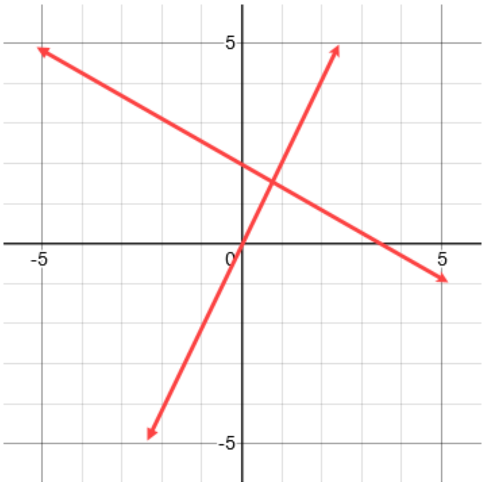




When perpendicular lines are in slope-intercept form their slopes are opposite (they are negative inverses of each other). In the first example above, on the left there are slopes of zero slope and undefined. They are opposites of each other. In the above right set of intersecting lines if two points are chosen and plotted it will show that their slopes are 1 and -1.

Example:

Taking two points from the first line (-2,3) and (0,2) the slope is:



$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2-3}{0--2} = \frac{-1}{+2} = -\frac{1}{2}$$

Next take two points from the second line (0,0) and (1,2), slope is:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2-0}{1-0} = \frac{2}{1} = 2$$

Two is the same as $\frac{2}{1}$ which is the negative inverse of $-\frac{1}{2}$, therefore, the two lines are perpendicular.

Look at some other slopes that are fractions. In the example above there is a whole number and a fraction for the slopes. The whole number becomes a fraction by placing it over 1, then comparing it to the other fraction.

The negative inverse of 2 is $-\frac{1}{2}$.

The negative inverse of -5 is $\frac{1}{5}$.

It is as simple as the first fraction being flipped and making sure one fraction is positive while the other is negative.

Note: If the fraction is a mixed number, make it an improper fraction: $2\frac{1}{2}$ *becomes* $\frac{5}{2}$.

Determine if the following lines are perpendicular from their slope-intercept equations.

$$y = 2x - 1 \text{ and } y = -\frac{1}{2}x + 7$$

1. Take the slope of the first equation and make it a fraction $2 = \frac{2}{1}$
2. Flip it.

$$\frac{2}{1} \text{ flipped becomes } \frac{1}{2}$$

3. Reverse the sign (since no sign is there it is positive).

$$-\frac{1}{2}$$

Because 2 is the negative inverse of $-\frac{1}{2}$ the lines are perpendicular.

Find the multiplicative negative inverse of:

J. -3 K. $\frac{1}{2}$ L. $-\frac{4}{9}$

Practice – Slope Intercept Format

For the following equations state the slope and the y-intercept

1. $y = 2x - 7$ slope: __________ y-intercept: _________
2. $y = -\frac{1}{3}x + 2$ slope: __________ y-intercept: _________
3. $y = \frac{3}{4}x + 1$ slope: __________ y-intercept: _________
4. $y = -x + -1$ slope: __________ y-intercept: _________
5. $y = 12x$ slope: __________ y-intercept: _________

Put the following equations into slope-intercept format and state the slope and the y-intercept.

Example: $4y - 4x = 16$

$$4y - 4x = 16$$
$$+4x \quad +4x$$
$$4y = 4x + 16$$

Isolate the variable

Make your y value equal to 1.

$$\frac{4y}{4} = \frac{4x}{4} + \frac{16}{4} \quad = y = x + 4$$

6. $2y+6x=12$; $y=$ ___ $x+$ ____ slope:___ y-intercept __

7. $x+2y=-1$; $y=$ ___ $x+$ ____ slope:___ y-intercept __

8. $-y=4x-2$; $y=$ ___ $x+$ ____ slope:___ y-intercept __

9. $-y-2=x$; $y=$ ___ $x+$ ____ slope:___ y-intercept __

Practice – Determining if Two Lines are Parallel or Perpendicular Given their Equations

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Determine if the following lines are parallel, perpendicular, or neither.

1. a. $y=2x+3$ slope = _______

   b. $y=-\frac{1}{2}x-2$ slope = _______ Parallel Perpendicular Neither

2. a. $y=4x$ slope = _______

   b. $y=4x+3$ slope = _______ Parallel Perpendicular Neither

3. a. $y=4x-2$ slope = _______

   b. $y=-4x+1$ slope = _______ Parallel Perpendicular Neither

4. a. $y=2x-1$ slope = _______

   b. $y=2x+12$ slope = _______ Parallel Perpendicular Neither

5. a. $y=-2x$ slope = _______

   b. $y=\frac{1}{2}x+8$ slope = _______ Parallel Perpendicular Neither